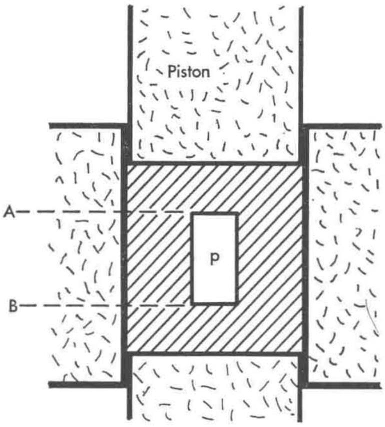

Fig. 11. Simplified model for piston-cylinder high pressure apparatus

in the assembly as well as the truly elastic compression of the parts. We therefore assume that all parts of the medium will have yielded and we consider the situation in the last pressure increment by which the final pressure is attained. During this increment, all parts of the medium are therefore assumed to be yielding plastically but without significant volume change, and we can then calculate the stresses in it using the theory of plasticity. The inside and outside diameters are taken to be 2a and 2b, respectively, and we use conventional cylindrical coordinates r, θ, z where z is directed along the apparatus axis. Following the same procedure as in Hoffman and Sachs (1953, p. 90–93) but using the above boundary conditions, it can be shown that the axial compressive stress component σ_z is given by

$$\sigma_z = p + \frac{\sigma_0}{\sqrt{3}}\left(\frac{1+3r^2/b^2}{\sqrt{1+3r^4/b^4}} + \coth^{-1}\sqrt{1+3a^4/b^4} - \coth^{-1}\sqrt{1+3a^4/b^4}\right)$$

if one assumes that yielding is controlled by the von Mises criterion, σ_0 being the uniaxial yield stress. The Tresca yield criterion is difficult to apply in this situation but in general can be expected to give a result not more than about 15 percent different.

The total force to be applied by the piston at the face A of the pressure cell is then

$$F = \pi a^2 p + \int_a^b \sigma_z \, 2\pi r \, dr$$

and so the nominal pressure p_n is

$$p_n = \frac{F}{\pi b^2} = \frac{a^2}{b^2} p + \frac{2}{b^2}\int_a^b \sigma_z r \, dr.$$

Substituting the above expression for σ_z and carrying out the integration gives

$$p = p_n - C\sigma_0$$

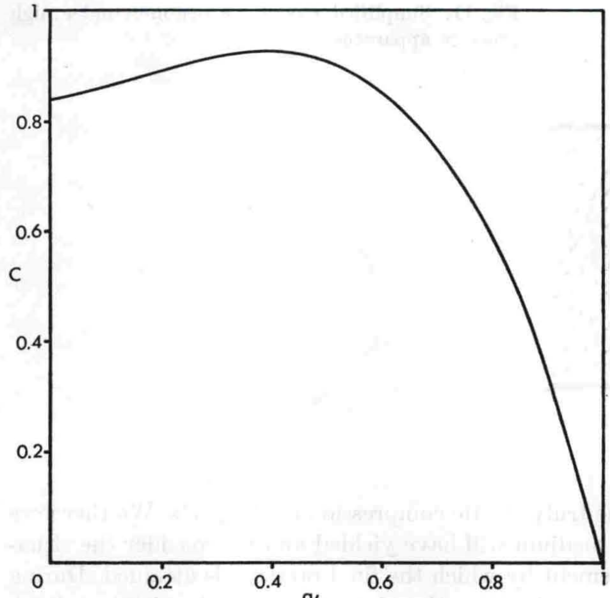

Fig. 12. Correction factor C as a function of diameter ratio a/b

where

$$C = \frac{1}{\sqrt{3}}\left[2 - \coth^{-1} 2 + \frac{a^2}{b^2}\left(\coth^{-1}\sqrt{1+3\frac{a^4}{b^4}} - \sqrt{1+3\frac{a^4}{b^4}}\right)\right].$$

Thus $C\sigma_0$ is the correction to be subtracted from the nominal pressure, where C is a geometrical factor, plotted as a function of a/b in Fig. 12, and σ_0 is the uniaxial compressive yield stress, which for a perfectly plastic material will be the same as the differential stress at yield in the high pressure tests reported in this paper. Only a slightly different result would be expected if the Tresca yield criterion were used.

This calculation can also be extended to take into account the change in bore diameter of the pressure vessel. Thus, if we take as boundary condition the radial displacement at the radius b to be $u_r = Kbp$ instead of zero (K is a constant involving the geometry of the pressure vessel and the compliance of its material), we now get

$$p = p_n - C'\sigma_0$$

where

$$C' = \frac{1}{\sqrt{3}}\left[2 - \coth^{-1} 2 + \frac{a^2}{\beta^2}\left(\coth^{-1}\sqrt{1+3\frac{a^4}{\beta^4}} - \sqrt{1+3\frac{a^4}{\beta^4}}\right)\right]$$

and

$\beta^2 = b^2\left(1 + 2K\frac{dp}{d\varepsilon_3}\right)$, ε_3 being the axial strain. That is, the expression for the correction is the same as before but with β substituted for b. Since K is positive and $\frac{dp}{d\varepsilon_3}$ negative, $\beta^2 < b^2$ always, and so the value of C' will correspond to a point to the right of the abscissa a/b in Fig. 12.